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NATURAL CONVECTIVE HEAT TRANSFER FROM A CYLINDER IN AN ENCLOSURE PARTLY FILLED WITH A POROUS MEDIUM

P.H. OOSTHUIZEN AND D. NAYLOR

Department of Mechanical Engineering, Queen's University, Kingston, Ontario, Canada

ABSTRACT

Heat transfer from a cylinder placed on the vertical centre-line of a square enclosure partly filled with a porous medium that is saturated with a fluid has been numerically studied. The cylinder is buried in the porous medium. The horizontal upper surface of the porous medium is separated from the rest of the enclosure by a horizontal impermeable barrier that is assumed to offer negligible resistance to heat transfer. The gap between the barrier and the top of the enclosure is filled with the same fluid as that with which the porous medium is saturated. The surface of the cylinder is at a uniform high temperature. The bottom and sides of the enclosure are assumed to be adiabatic while the horizontal upper surface of the enclosure is assumed to be kept at a uniform low temperature. The natural convective flows that occur in the porous medium and in the fluid layer above the barrier have been assumed to be steady, laminar, two-dimensional and symmetrical about the vertical centre-line of the enclosure. Fluid properties have been assumed constant except for the density change with temperature which gives rise to the buoyancy forces. The governing equations have been expressed in dimensionless form and solved using a finite element procedure. Results have been obtained for a Prandtl number of 0.7 for a wide range of the governing parameters. The main aim of the study was to determine how the mean heat transfer rate from the cylinder is affected by the size of the fluid gap at the top of the enclosure. The effect of this gap size has been related to changes in the flow pattern in the porous and fluid regions.

KEYWORDS Natural convection Porous media Enclosures

NOMENCLATURE

c = Specific heat
D' = Diameter of cylinder
Da = Darcy number
H = H'/D'
H' = Thickness of fluid layer
g = Gravitational acceleration
K = Permeability of porous medium
k = Thermal conductivity
k_r = k_p/k_f
L_1 = L'_1/D'
L'_1 = Distance from bottom of enclosure to cylinder centre-line
L_2 = L'_2/D'
L'_2 = Distance from bottom of enclosure to cylinder centre-line
Nu = mean Nusselt number for cylinder based on D'
n = n'/W'
n' = Co-ordinate measured normal to surface
Pr = Prandtl number
p' = Pressure
Ra = Rayleigh number based on D'
S = S'/D'
S' = Size of enclosure
T = Dimensionless temperature
T' = Temperature
T'_H = Temperature of cylinder surface
T'_C = Temperature of cold top wall
u' = Velocity component in x' direction
v' = Velocity component in y' direction
W' = Width of enclosure
x = Dimensionless x' co-ordinate
x' = Horizontal co-ordinate position
y = Dimensionless y' co-ordinate
y' = Vertical co-ordinate position
β = Coefficient of thermal expansion
ν = Kinematic viscosity
ρ = Density
ψ = Dimensionless stream function
ψ' = Stream function
ω = Dimensionless vorticity
ω' = Vorticity

Subscripts
f = Fluid properties
p = Porous media properties

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INTRODUCTION

In some building service situations, a pipe carrying hot water passes through an enclosure formed by structural components of the building. Most of the sides of the enclosure so formed are usually effectively adiabatic but one surface may be exposed to the external ambient environment and, as a result, will be at a low temperature. In such situations, the enclosure is often partly filled with insulation to reduce the heat transfer rate from the pipe. A typical although idealized such situation has been considered in the present study, this being shown schematically in *Figure 1*. Thus, flow in a square enclosure has been considered in the present study. The bottom and sides of this enclosure are assumed to be adiabatic while the upper surface of the enclosure is assumed to be at a uniform low temperature T'_C. A cylinder is buried in the saturated porous insulating material contained in the lower part of the enclosure, the upper surface of this porous medium being separated from the rest of the enclosure by a horizontal impermeable barrier that is assumed to offer negligible resistance to heat transfer. The surface of the cylinder has been assumed to be kept at a uniform high temperature, T'_H. The natural convective flow that occurs in the porous medium and in the fluid layer above the barrier has been assumed to be two-dimensional and symmetrical about the vertical centre-line.

The main aim of the present study was to determine how the heat transfer rate from the cylinder is affected by the size of the air gap for a given enclosure size. The results have application in situations where the heat transfer has to be reduced but cost considerations require that as little insulation material as possible be used.

The flow and heat transfer in enclosures that are partly filled with a fluid and partly filled with a porous medium have previously been considered by a number of workers both for the case where there is no barrier between the layers and for the case where there is an impermeable barrier between the layers. Typical of these studies are those of Poulikakos and Bejan[1], Lauriat and

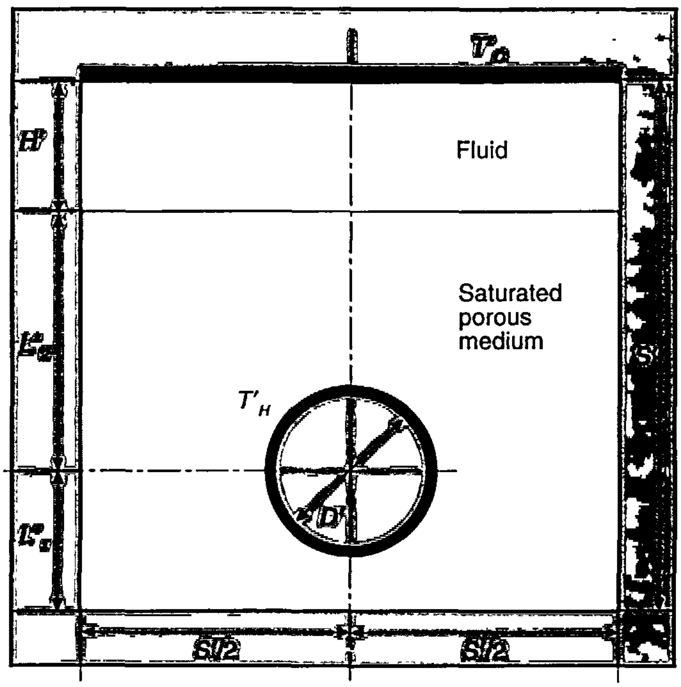

Figure 1 Situation considered in present study

Mesguich[2], Beckermann *et al.*[3], Arquis *et al.*[4], Oosthuizen and Paul[5-8], Tong and Subramanian[9], Tong *et al.*[10], Song and Viskanta[11] and Naylor and Oosthuizen[12]. These studies were all essentially concerned with flow in rectangular enclosures with all or part of the walls heated or cooled. No studies appear to be available of heat transfer from bodies in an enclosure which is partly filled with a porous medium.

GOVERNING EQUATIONS AND SOLUTION PROCEDURE

It has been assumed that the flow is steady, laminar and two- dimensional and that fluid properties are constant except for the density change with temperature which gives rise to the buoyancy forces, this being treated using the Boussinesq approach. The usual Darcy assumptions have then been adopted in the porous layer, except that the viscous shear stress term, i.e. the Brinkman term, has been retained although the inertia term has been neglected. The flow has been assumed to be symmetrical about the vertical centre-line and only the solution for one half of the enclosure has been found, the solution domain, therefore, being as shown in *Figure 2*. In order to check whether the assumption of symmetry was valid, a limited number of calculations were undertaken in which the flow in the entire enclosure was considered. In all of these cases it was found that the flow was indeed symmetrical about the vertical centre-line of the enclosure.

The solution has been obtained in terms of the stream function and vorticity defined, as usual, by:

$$u' = \frac{\partial \psi'}{\partial y'}, v' = -\frac{\partial \psi'}{\partial x'}, \omega' = \frac{\partial v'}{\partial x'} - \frac{\partial u'}{\partial y'}. \tag{1}$$

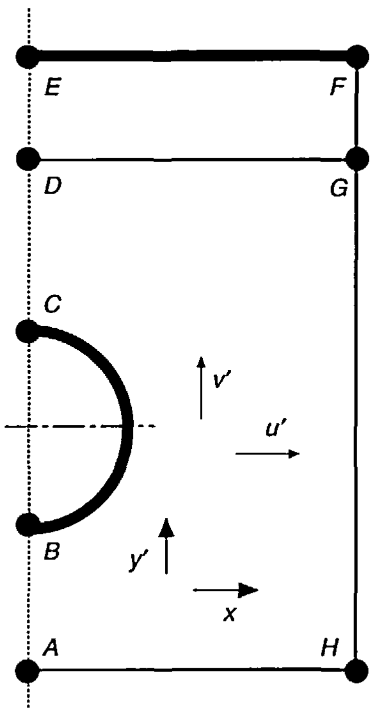

Figure 2 Boundary segments and co-ordinate system used

The prime (′) denotes a dimensional quantity. In the porous layer, the velocity is, of course, the superficial or Darcian mean velocity.

The following dimensionless variables have then been defined:

$$\psi = \psi'/\alpha_f, \omega = \omega' D'^2/\alpha_f, T = (T - T'_C)/(T'_H - T'_C) \tag{2}$$

where $\alpha_f = k_f/\rho_f c_f$ and where the subscript f denotes fluid properties. The cold wall temperature, T'_C, has been taken as the reference temperature. The co-ordinate system used is shown in *Figure 2*.

In terms of these dimensionless variables, the governing equations for the porous medium are:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = -\omega \tag{3}$$

$$\left(\frac{\nu_p}{\nu_f}\right)\left(\frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y^2}\right) - \frac{\omega}{Da}$$

$$= -Ra\frac{\partial T}{\partial x} \tag{4}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} - \left(\frac{k_p}{k_f}\right)\left(\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}\right) = 0. \tag{5}$$

Similarly, the dimensionless governing equations for the fluid layer are:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = -\omega \tag{6}$$

$$\left(\frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y^2}\right) - \frac{1}{Pr}\left(\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y}\right)$$

$$= -Ra\left(\frac{\partial T}{\partial x}\cos\phi + \frac{\partial T}{\partial y}\sin\phi\right) \tag{7}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} - \left(\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}\right) = 0. \tag{8}$$

In these equations, Ra is the Rayleigh number based on the cylinder diameter and defined as usual by:

$$Ra = \beta g D'^3(T'_H - T'_C)/\alpha_f \nu_f. \tag{9}$$

The boundary conditions on the solution are: on all walls i.e. on BC, EF, FH and HA in *Figure 2*:

$$\psi = 0, \frac{\partial\psi}{\partial n} = 0$$

On BC in *Figure* 2:

$$T = 1$$

On EF in *Figure* 2:

$$T = 0$$

On FH and AH in *Figure 2*:

$$\frac{\partial T}{\partial n} = 0 \qquad (10)$$

On AB and CE in *Figure* 2:

$$\omega = 0, \frac{\partial T}{\partial x} = 0$$

where n is the co-ordinate measured normal to the wall surface considered.

On the impermeable partition between the porous and fluid layers, i.e. on DG in *Figure 2*, which, by assumption, offers no resistance to heat resistance to heat transfer, the following conditions apply:

$$\psi = 0, \frac{\partial \psi}{\partial y} = 0$$

$$\frac{\partial T}{\partial y}\Big|_i = \frac{\partial T}{\partial y}\Big|_{i+1}\left(\frac{\alpha_{i+1}}{\alpha_i}\right)$$

where the subscripts i and $i + 1$ refer to conditions on the two sides of the partition.

The above dimensionless equations, subject to the boundary conditions, have been solved using a Galerkin based finite element procedure, the non-linear finite-element equations being solved using an iterative procedure. The solutions for the porous medium and fluid layers were obtained simultaneously using the matching conditions across the impermeable barrier. Nodal points were selected to lie along the barrier in both the porous medium and the fluid regions so that the elements were either entirely in the porous medium or entirely in the plain fluid. The numerical procedure adopted has previously been successfully used to study many enclosure flow problems involving either pure fluids or porous media or both.

Extensive grid independency testing was undertaken, solutions for the same set of parameters for between approximately 1,500 and 3,500 elements being obtained for a number of parameter sets. Very little dependency of the mean Nusselt number on the number of elements was found over this range of elements. For example, for one typical set of parameters, mean Nusselt numbers of between 1.50 and 1.48 were found over this range of numbers of elements. The results presented in the figures discussed below were all obtained using approximately 2,500 elements and with this number of elements the difference between the calculated Nusselt number and the value deduced from the grid-dependency results for an infinite number of elements was always less than 1 per cent.

No indication of the existence of multiple solutions was found for the range of parameters covered in the present study.

RESULTS AND DISCUSSION

The solution has the following parameters:

- the Rayleigh number, *Ra*;
- the Darcy number, *Da*;
- the Prandtl number, *Pr*;
- the dimensionless size of the enclosure *S*;
- the dimensionless vertical position of the cylinder, L_1;
- the dimensionless size of the fluid layer, *H*;
- the conductivity ratio, k_r;
- the viscosity ratio.

Results have only been obtained for air, i.e. for a Prandtl number of 0.7. The viscosity ratio has been taken as 1 which appears, on the basis of available experimental results, to be a reasonably good assumption. With the Prandtl number fixed and the viscosity ratio taken as 1, the governing parameters reduce to the Rayleigh number, the Darcy number, the conductivity ratio and the geometrical parameters. Rayleigh numbers of between 10,000 and 100,000, Darcy numbers of between 0.0002 and 0.02, conductivities ratios of between 1 and 3 and dimensionless enclosures sizes between 6 and 8 have been considered in the present study. It should be noted that the Rayleigh and Darcy numbers used here are based on the cylinder diameter not on the enclosure size. It should also perhaps be noted that common insulating materials have conductivity ratios of roughly between 1.3 and 2.0.

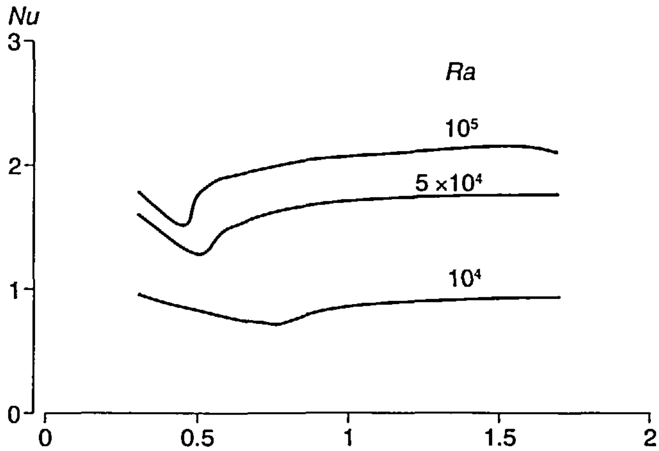

Figure 3 Variation of mean cylinder Nusselt number with dimensionless fluid layer thickness for various values of the Rayleigh number for $Da = 0.002$, $k_r = 2$, $S = 4$ and $L_1 = 1$

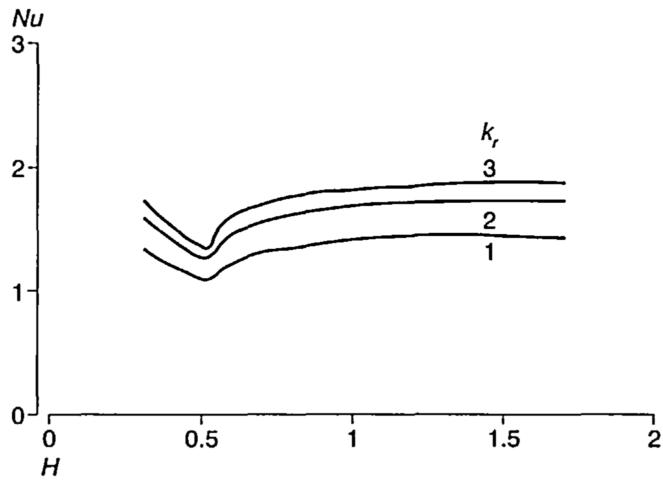

Figure 4 Variation of mean cylinder Nusselt number with dimensionless fluid layer thickness for various values of the conductivity ratio for $Ra = 50{,}000$, $Da = 0.002$, $S = 4$ and $L_1 = 1$

The main result considered here is the mean heat transfer rate from the cylinder. This mean heat transfer rate has been expressed in terms of a mean Nusselt number based on the cylinder diameter, D' and the overall temperature difference, $(T'_H - T'_C)$.

Typical variations of mean Nusselt number, *Nu*, with dimensionless fluid layer size, *H*, are shown in *Figures 3* to *7*. It will be seen that, in all cases, as *H* increases the mean Nusselt number initially decreases. It then passes through a minimum before rising sharply with further increase in *H* and then reaches a near constant value at larger *H* values. In all cases, then, there is an *H* value that gives a minimum heat transfer rate from the cylinder. The reason for this form of behaviour can be understood by considering the changes in the flow in the enclosure with changing *H*. Typical streamline and isotherm patterns for various values of *H* for fixed values of the other parameters are shown in *Figure 8*. It will be seen that at small values of *H*, there is no motion in

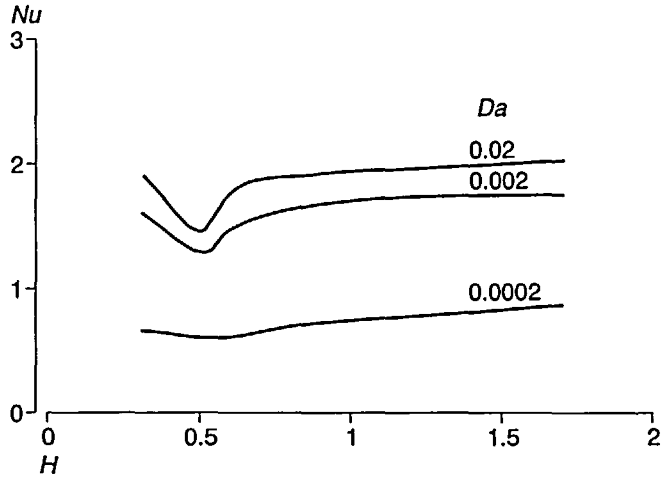

Figure 5 Variation of mean cylinder Nusselt number with dimensionless fluid layer thickness for various values of the Darcy number for $Ra = 50{,}000$, $k_r = 2$, $S = 4$ and $L_1 = 1$

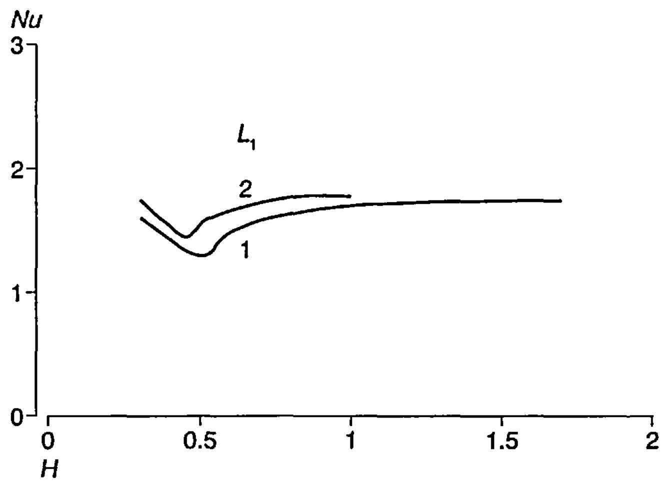

Figure 6 Variation of mean cylinder Nusselt number with dimensionless fluid layer thickness for two values of the dimensionless cylinder position for $Ra = 50{,}000$, $Da = 0.002$, $S = 4$ and $k_r = 2$

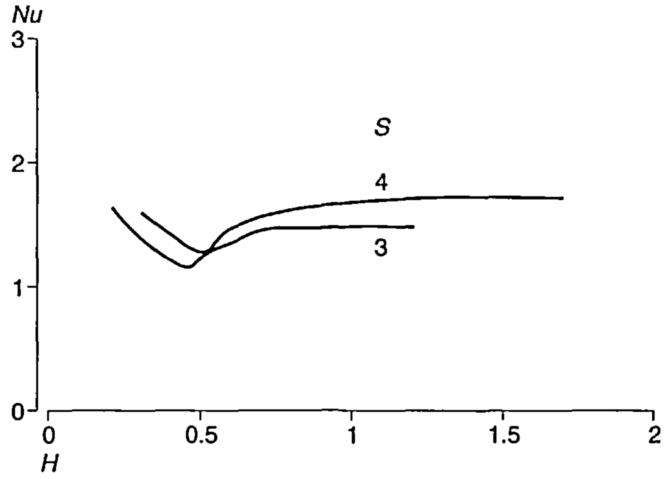

Figure 7 Variation of mean cylinder Nusselt number with dimensionless fluid layer thickness for two values of the dimensionless enclosure size for Ra = 50,000, Da = 0.002, k_r = 2 and L_1 = 1.

the fluid layer. However, the fluid layer is unstable, the barrier being at a higher temperature than the upper surface. At a critical value of H, therefore, motion develops in this fluid layer. Initially, this motion consists of two or three vortices but with increasing H it develops into a single vortex rotating in the same direction as the vortex in the porous medium. The conditions at which the motion begins in the fluid layer cannot be deduced from available results for flow in unstable fluid layers which assume that the upper and lower surfaces are at uniform temperatures. In the present case, the temperature of the barrier, which forms the lower boundary of the fluid layer, is not uniform and depends on the motion in the fluid layer. The rapidity of the growth of the fluid motion with increasing H can be seen by considering the results given in *Figure 9*. This shows streamline and isotherm patterns for H values of 0.46, 0.48, 0.5 and 0.6. for fixed values of the other parameters. The fluid flow will be seen to develop over a small range of H values and marked changes in the fluid flow pattern will be seen to occur with very small changes in H. Even when the fluid motion in the porous medium is very weak, i.e. when the Darcy number is small, large changes occur in the flow pattern in the fluid layer. This is illustrated by the results given in *Figure 10*.

Returning to a consideration of the variation of Nusselt number with H, it follows from the discussion given above that at small values of H the only fluid motion is in the porous medium and that the fluid layer offers a relatively higher resistance to heat transfer than the porous layer. Increasing the thickness of the fluid layer then increases the the overall thermal resistance of the system leading to a decrease in the heat transfer rate. Furthermore, in most of the cases considered, the porous medium has a higher effective thermal conductivity than the fluid layer. When there is no motion in the fluid layer, this makes the thermal resistance of the fluid layer very much lower than that of the porous layer. This accentuates the decrease in mean heat transfer rate with increasing fluid layer thickness at small values of H. Once the motion starts in the fluid layer, however, the thermal resistance of the fluid layer decreases rapidly, leading to a decrease in the overall thermal resistance of the system and so leading to the observed increase in the mean heat transfer rate. Considering the results given in *Figure 3*, it will be seen that the value of H at which the minimum in the mean Nusselt number variation occurs increases with decreasing Rayleigh number, the variation being shown in *Figure 11*. It will further be seen that at the lowest Rayleigh number considered, when the motion in both the porous or fluid layers is very weak, the variation of Nusselt number with H is small. From *Figure 4* it will be seen that the conductivity ratio has

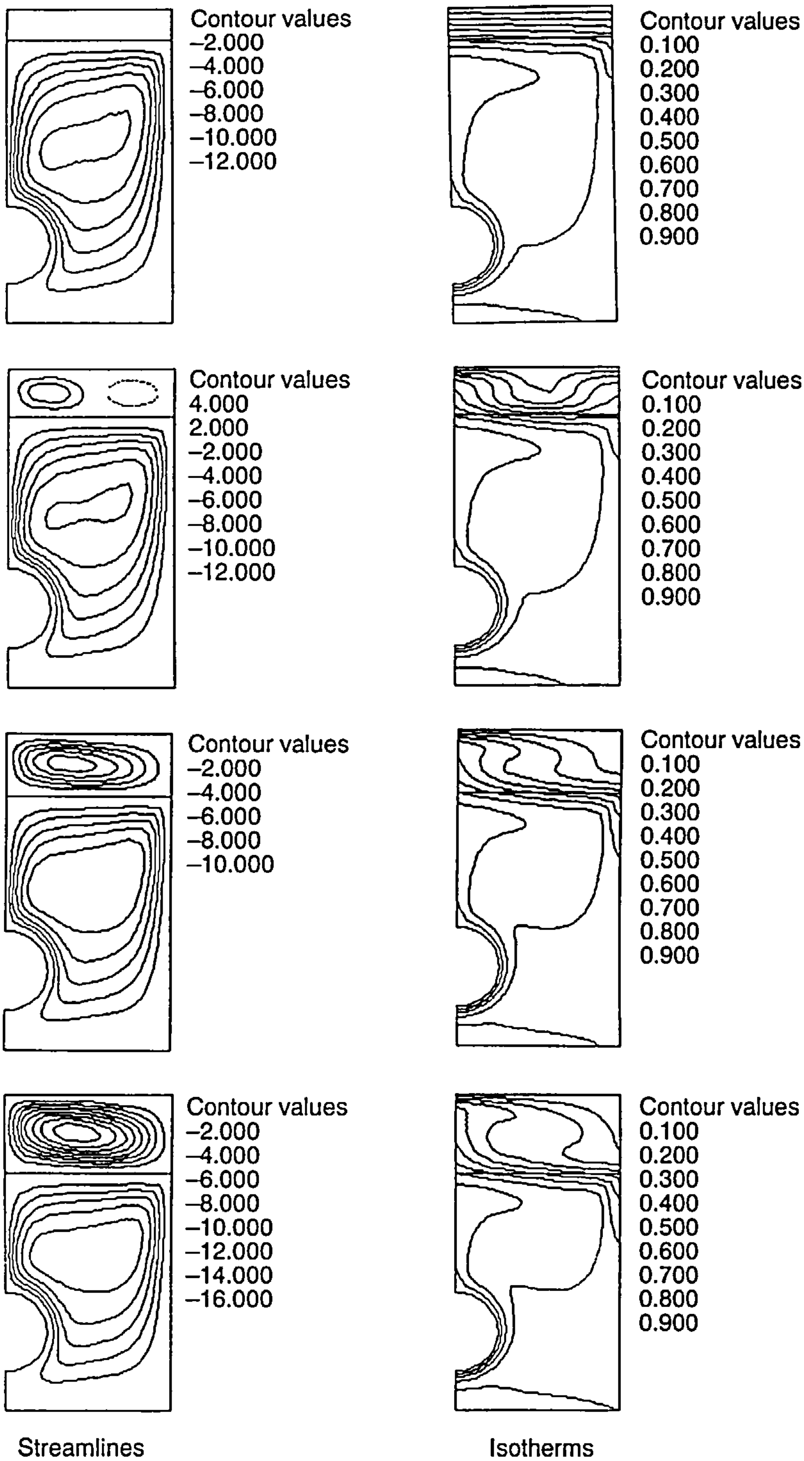

Figure 8 Streamline and isotherm patterns for $Ra = 50{,}000$, $Da = 0.002$, $S = 4$ and $L_1 = 1$ for dimensionless fluid layer thicknesses, H of, from the top, 0.4, 0.6, 0.8 and 1

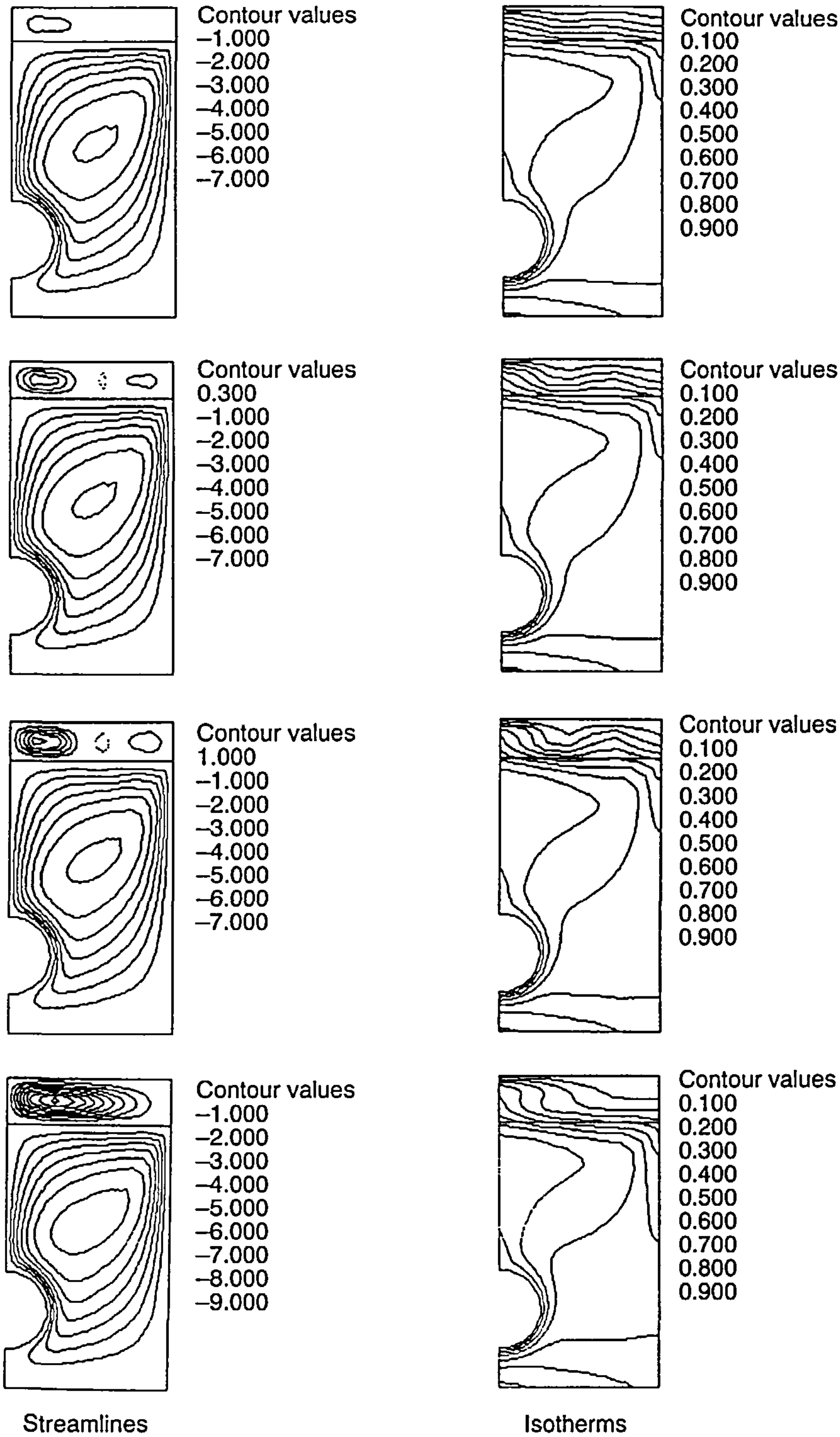

Figure 9 Streamline and isotherm patterns for Ra = 100,000, Da = 0.002, S = 4 and L_l = 1 for dimensionless fluid layer thicknesses, H of, from the top, 0.46, 0.48, 0.5 and 0.6

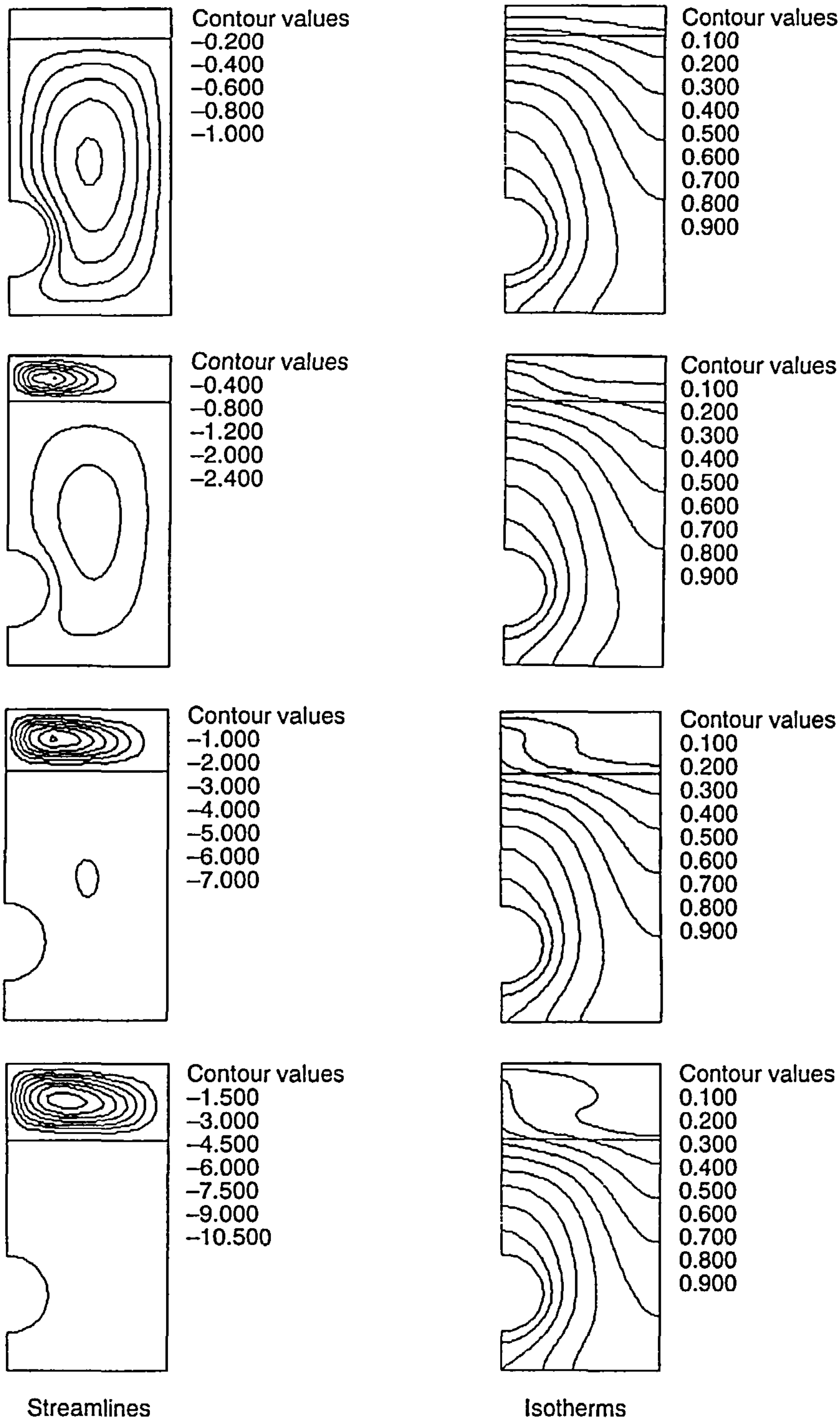

Figure 10 Streamline and isotherm patterns for Ra = 50,000, Da = 0.0002, S = 4 and L_1 = 1 for dimensionless fluid layer thicknesses, H of, from the top, 0.4, 0.6, 0.8 and 1

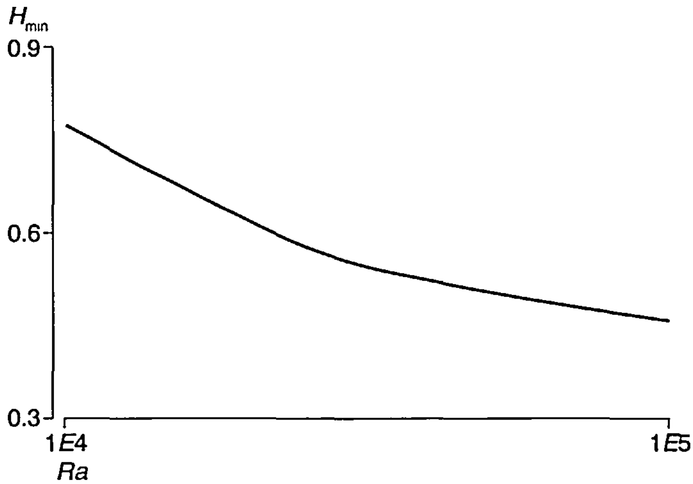

Figure 11 Variation of dimensionless fluid layer thickness at minimum Nusselt number with Darcy number for $Ra = 50{,}000$, $k = 2$, $S = 4$ and $L_1 = 1$.

little effect on the value of H at which the minimum Nusselt number occurs. *Figure 5* shows the effect of Darcy number on the Nusselt number variation with H. The value of H at which the minimum Nusselt number occurs increases somewhat with decreasing Darcy number, the variation being shown in *Figure 12*. It will further be seen that at the smaller Darcy number considered, when there is relatively little motion in the porous medium layer, there is relatively little change in Nusselt number with H. This is because in this case for the range of H values considered the porous medium is the major source of thermal resistance and the changes in the flow in the fluid layer consequently have a weak effect on the heat transfer rate. *Figure 6* illustrates the effect of the dimensionless distance of the cylinder from the bottom of the enclosure on the Nusselt number variation with H. The effect will be seen to be relatively weak, the value of H at which the minimum Nusselt number occurs decreasing somewhat with increasing distance.

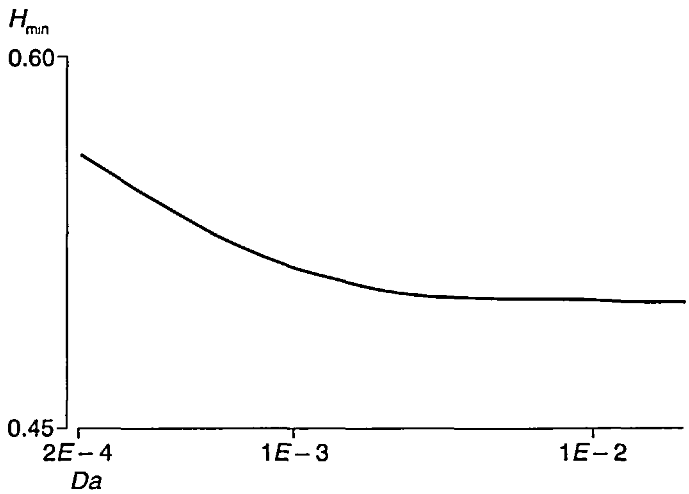

Figure 12 Variation of dimensionless fluid layer thickness at minimum Nusselt number with Rayleigh number for $Da = 0.002$, $k = 2$, $S = 4$ and $L_1 = 1$

Last, *Figure 7* illustrates the effect of changes in the dimensionless enclosure size on the variation of Nusselt number with H. The effect of S on the variation will also be seen to be relatively weak for the range of values considered. The value of H at which the minimum Nusselt number occurs will be seen to decrease somewhat with decreasing S.

CONCLUSIONS

In all cases considered, it has been found that there is a dimensionless fluid layer thickness that gives a minimum mean Nusselt number for a given situation. This minimum has been shown to arise because when the dimensionless fluid layer thickness is small, there is no motion in the fluid layer leading to a decrease in the mean heat transfer rate with increasing dimensionless fluid layer thickness. However, once the dimensionless fluid layer thickness reaches a certain critical size, motion rapidly develops in the fluid layer leading to an increase in the mean heat transfer rate with further increase in the dimensionless fluid layer thickness. These two effects together lead to the minimum in the Nusselt number variation.

The effect of the various governing parameters on the value of the dimensionless fluid layer thickness at which the minimum Nusselt number occurs has been explored.

ACKNOWLEDGEMENTS

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